Holdup in Vertical Upward Slug Flow

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Several slug flow models for holdup have been developed for the flow of long, round-nosed gas bubbles in liquid (*10, 14*). The model recently proposed by Brown and Govier (*2*) is of particular interest because it accounts for the effect of the liquid viscosity. The model is defined by the equations

$$\frac{V_b}{V_b{}^0} = f\left(C_0 \frac{V_{av}}{V_b{}^0}\right) \qquad (1)$$

$$V_b{}^0 = 0.496\sqrt{gR}\sqrt{1 - \frac{\delta_0}{R}} \qquad (2)$$

$$\delta_0 = \frac{-1 + \sqrt{1 + 2NR}}{N} \qquad (3)$$

$$C_0 = \frac{1}{(1 - \epsilon_0)^2} \qquad (4)$$

Equation (1) was tested by Brown and Govier with the experimental data of Govier and co-workers (*3, 7, 8*) and of Hughmark (*11*) for vertical upward air-liquid flow. The results indicate that the data of Govier and co-workers for 0.63- to 1.5-in. diameter tubes are represented by a single line but that the 2.5-in. diameter tube data fall slightly below the line for the smaller diameters. The Hughmark data show a trend with liquid viscosity. Thus, it appears that Equation (1) can be applied for slug flow if the factor f is empirically correlated as a function of tube diameter and liquid viscosity.

CORRELATION

Phase density did not appear as a variable in the Brown and Govier work because only air-liquid systems were considered. A general correlation would be expected to apply to high-pressure steam-water data and even to extend to slug flow data for water-oil systems. Such a correlation was developed from experimental vertical upward holdup data.

For tube diameters from 0.427 to 1.5 in. and for $x > 10$:

$$\ln f = [0.1158 - 0.034\,(-2.83 + \ln x)\,(0.341 + \ln \mu_L')]\left(\frac{\rho/\rho_g}{750}\right)^{0.02} \qquad (5)$$

and for $x < 10$:

$$\ln f = [0.045 - 0.0236\,(-2.94 + \ln x)\,(5.09 + \ln \mu_L')]\left(\frac{\rho/\rho_g}{750}\right)^{0.02} \qquad (6)$$

For tube diameters from 2 to 2.5 in. and for $x < 10$:

$$\ln f = (0.556 - 0.121 \ln x - 0.031 \ln \mu'_L \left(\frac{\rho/\rho_g}{750}\right)^{0.02} \qquad (7)$$

and for $x < 10$:

$$\ln f = (0.6385 - 0.1565 \ln x - 0.031 \ln \mu'_L)\left(\frac{\rho/\rho_g}{750}\right)^{0.02} \qquad (8)$$

The factor f was found to vary with the viscosity of the heavy phase if this viscosity was between 1 and 25 centipoise. If the heavy phase viscosity was less than 1 centipoise, the data fitted the 1-centipoise line. If the heavy phase viscosity was greater than 25 centipoise, the data fitted the 25-centipoise line. f is apparently independent of the light phase viscosity because the liquid-liquid data are correlated by the equations for f even though the light phase viscosity varies from 0.936 to 150 centipoise.

Figure 1 compares experimental and calculated holdups for several systems, and Table 1 compares the experimen-

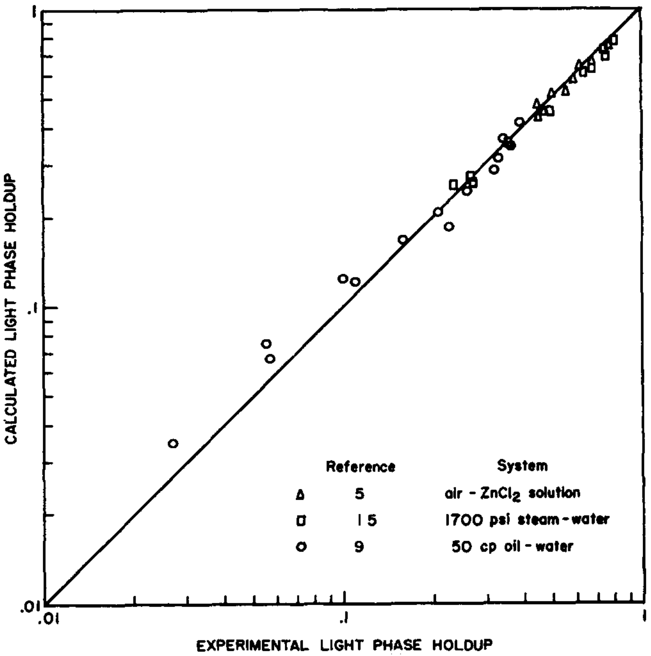

Fig. 1. Comparison of experimental and calculated data.

TABLE 1. COMPARISON OF CALCULATED AND EXPERIMENTAL LIQUID HOLDUP DATA

Reference	Pipe I.D., in.	System	Av. pressure, lb./sq. in. abs.	No. of runs	Av. absolute deviation, % Reference *12*	This paper
1	0.427	Air-water	atm.	36	16.3	8.3
6	0.5	Air-water	atm.	9	24	10
4	0.71	Argon-water	213	4	11.2	5.2
13	0.872	Steam-water	atm.	20	16.8	13.3
4	0.985	Argon-water	213	20	8.7	8
4	0.985	Argon-ethanol	213	6	4.3	2.8
7	1.025	Air-water	36	91	8.2	7.1
11	1.059	Air-water	atm.	114	7	4.5
11	1.059	Air-sodium carbonate solution	atm.	61	6.6	5.6
11	1.059	Air-Varsol	atm.	55	9.3	8.7
11	1.059	Air-Oil Blend 1	atm.	54	5.2	7.7
11	1.059	Air-Oil Blend 2	atm.	45	9.2	3.3
11	1.059	Air-Triclene	atm.	22	6.9	9.3
15	1.18	Steam-water	284	75	6.5	5.8
15	1.18	Steam-water	1,700	33	12.2	5
8	1.5	Air-water	36	39	7.9	9
6	2	Air-water	atm.	31	18.7	7
5	2	Air-water	atm.	31	17.1	7.4
5	2	Air-zinc chloride solution	atm.	31	13.7	5.5
5	2	Air-22 centipoise glycerol	atm.	42	12.2	4.5
5	2	Air-109 centipoise glycerol	atm.	41	4	4.5
5	2	Air-150 centipoise glycerol	atm.	36	8.3	4.2
8	2.5	Air-water	36	31	18.7	7
9	1.038	0.936 centipoise oil-water		21	19*	7.9*
9	1.038	20 centipoise oil-water		22	26*	8.5*
9	1.038	150 centipoise oil-water		12	31.5*	8*

* Oil holdup.

tal holdup data with that calculated from Equations (1) to (8) and with an earlier correlation (*12*) that is applicable to slug flow. This comparison shows that both correlations give about the same absolute average deviation for gas-liquid systems with tube diameters of the order of 1 in. The new correlation is better for the small diameter, 2- to 2.5-in. diameter data and the liquid-liquid data.

Brown and Govier show data for all flow regimes plotted with Equation (1). Analysis of the experimental data shows that only data in the slug flow regime is adequately predicted by the model.

SUMMARY

Empirical correlations are presented for the factor used by Brown and Govier in their model for vertical upward slug flow holdup. Calculated values of liquid holdup for 927 gas-liquid runs show an average deviation of 6.7% with tube diameters from 0.427 to 2.5 in. Fifty-five runs for water-oil slug flow in a 1-in. diameter tube show an average absolute deviation of 8% between calculated and experimental values for oil holdup.

NOTATION

- C_0 = area ratio of tube to a Taylor bubble in stagnant liquid
- f = factor
- g = acceleration of gravity, ft./sec.^{2}
- N = dimensional property parameter, $(14.5\, \rho^2 g/\mu^2)^{1/3}$, ft.$^{-1}$
- R = tube radius, ft.
- V_b = bubble velocity, ft./sec.
- $V_b{}^0$ = bubble velocity in stagnant liquid, ft./sec.
- V_{av} = average liquid velocity ahead of bubble, relative to tube, ft./sec.
- x = $C_0\,(V_{av}/V_b{}^0)$

Greek Letters

- δ_0 = equilibrium film thickness for a Taylor bubble in stagnant liquid, ft.
- μ = viscosity of heavy phase, lb.$_m$/(ft.)(sec.)
- μ'_L = viscosity of heavy phase, centipoise
- ϵ_0 = equilibrium relative film thickness for a Taylor bubble in stagnant liquid, δ_0/R
- ρ = density of heavy phase, lb.$_m$/cu.ft.
- ρ_G = density of light phase, lb.$_m$/cu.ft.

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